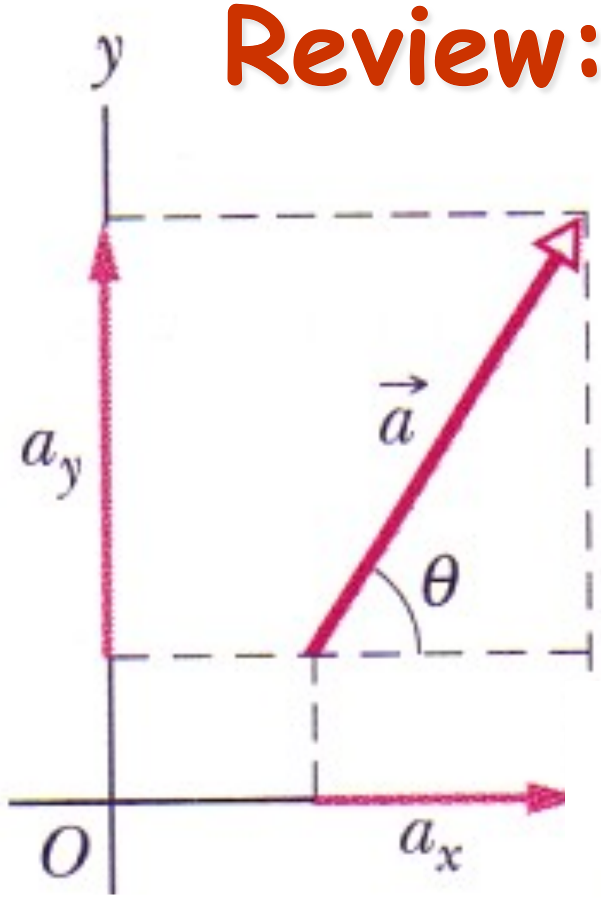
Chapter 3: 2D Kinematics
Thursday January 22nd

- 1st Mini Exam (25 minutes)
- Chapter 3: Motion in 2D and 3D
  - Short Review
  - Review: Projectile motion
  - More example problems
  - Range of a projectile
- Uniform Circular Motion (if time)
  - Centripetal acceleration

Reading: up to page 44 in the text book (Ch. 3)

Review: Components of vectors

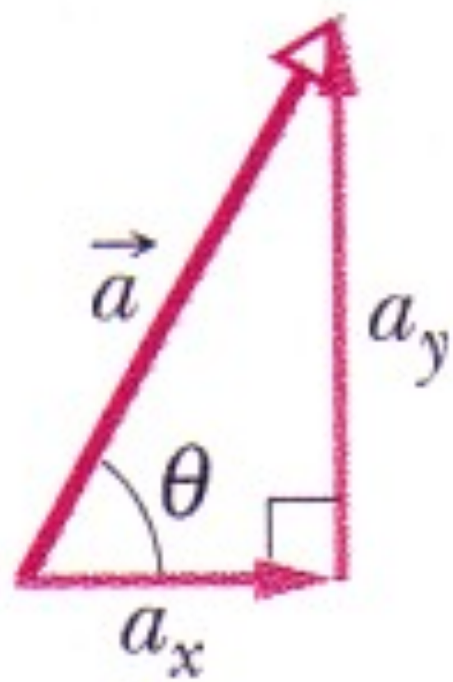


Resolving vector components

$$a_x = a\cos\theta$$

$$a_y = a\sin\theta$$

The inverse process

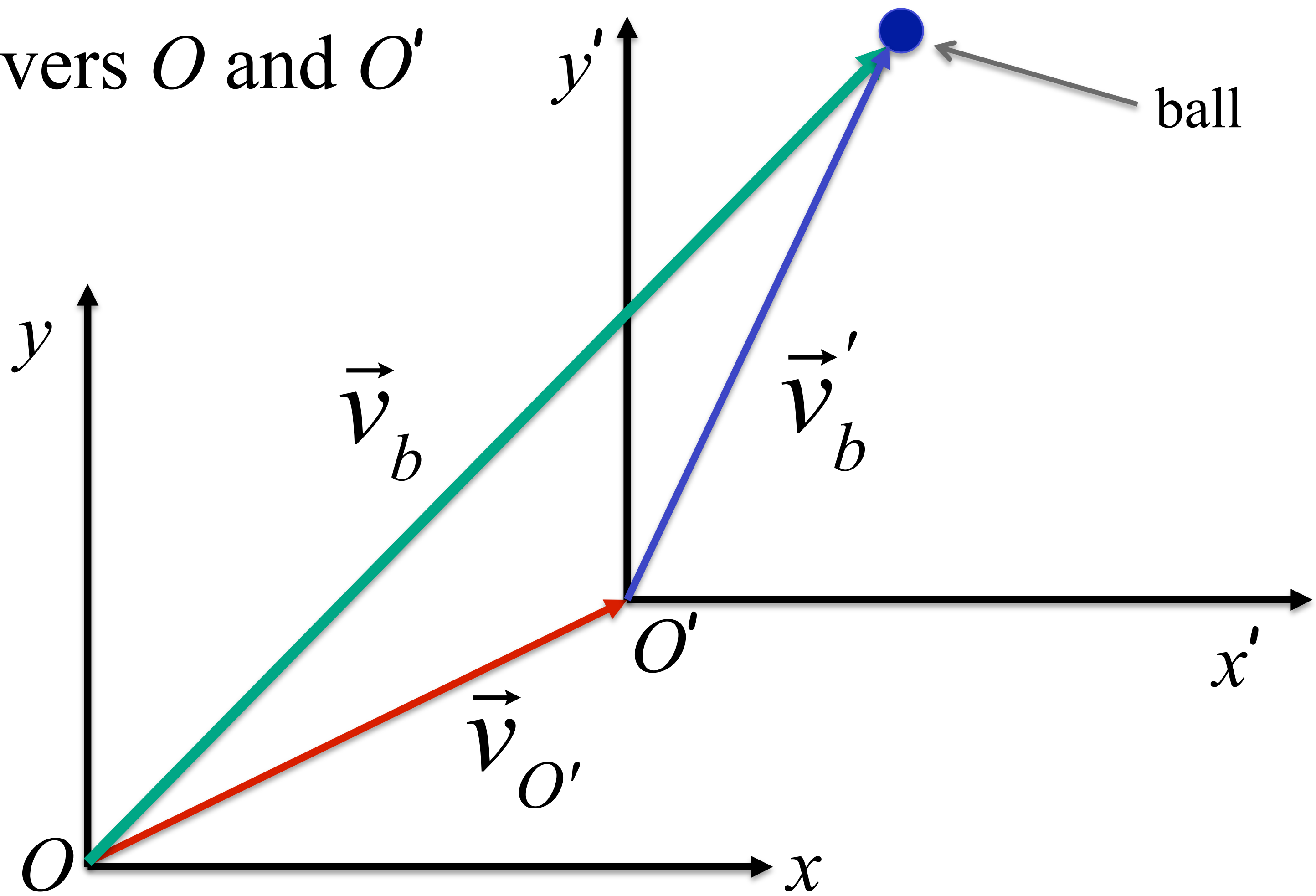


$$a = \sqrt{a_x^2 + a_y^2}$$

$$\tan\theta = \frac{a_y}{a_x}$$

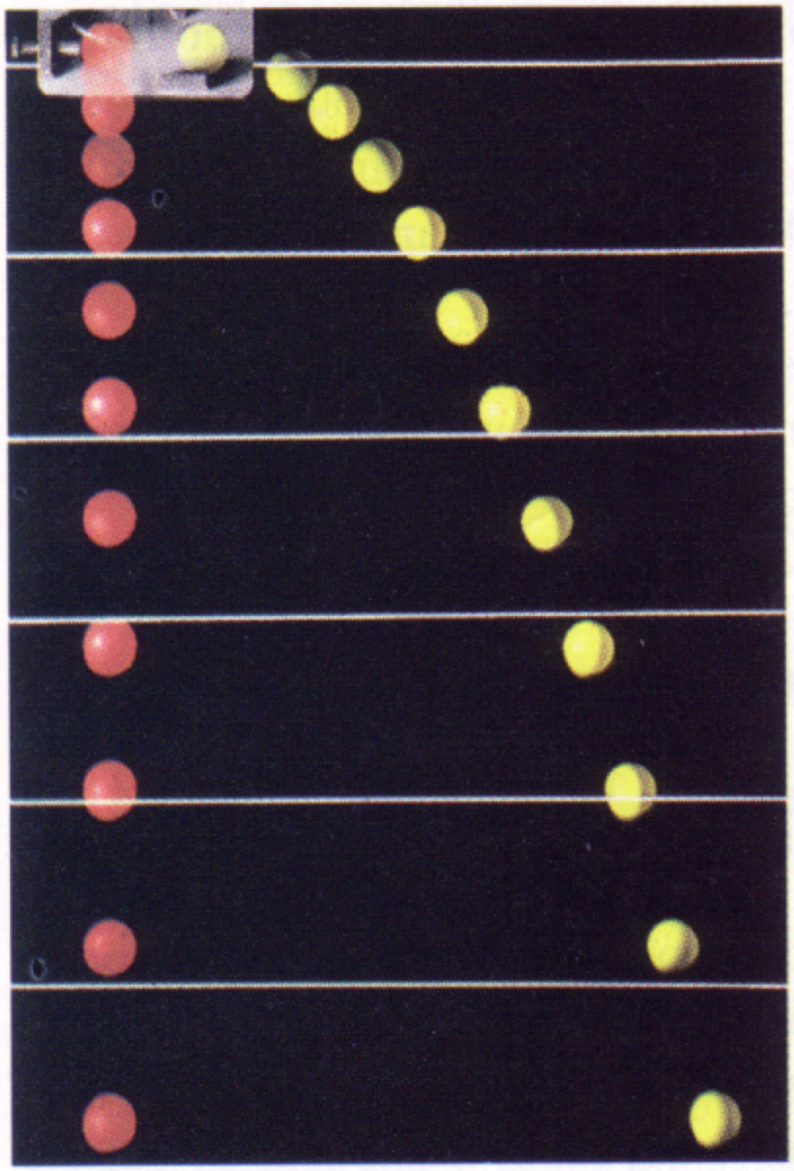
Review: Relative Motion

Two observers O and O'



Velocity of ball observed in O: $\vec{v}_b = \vec{v}_b' + \vec{v}_{O'}$

Projectile motion



- This series of photographic images illustrates the fact that vertical motion is unaffected by horizontal motion, i.e., the two balls accelerate downwards at the same constant rate, irrespective of their horizontal component of motion.
- In all of the projectile motion problems that we will consider, we shall assume that the only acceleration is due to gravity ($a=-g$) which acts in the $-y$ direction.

Equations of motion for constant acceleration

Equation number	Equation	Missing quantity
3.8	$\vec{v} = \vec{v}_0 + \vec{a}t$	$(\vec{r} - \vec{r}_0)$
3.9	$\vec{r} - \vec{r}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$	$\vec{v}$
	$v^2 = v_0^2 + 2\vec{a}\cdot(\vec{r} - \vec{r}_0)$	t
	$(\vec{r} - \vec{r}_0) = \frac{1}{2}(\vec{v}_0 + \vec{v})t$	$\vec{a}$
	$(\vec{r} - \vec{r}_0) = \vec{v}t - \frac{1}{2}\vec{a}t^2$	$\vec{v}_0$

Important: equations apply ONLY if acceleration is constant.

Equations of motion for constant acceleration

These equations work the same in any direction, e.g., along x, y or z.

Equation number	Equation	Missing quantity
2.7	$v_x = v_{0x} + a_x t$	$x - x_0$
2.10	$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$	v_x
2.11	$v_x^2 = v_{0x}^2 + 2a_x (x - x_0)$	t
2.9	$x - x_0 = \frac{1}{2}(v_{0x} + v_x) t$	a_x
	$x - x_0 = v_x t - \frac{1}{2} a_x t^2$	v_0

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Equations of motion for constant acceleration

These equations work the same in any direction, e.g., along x, y or z.

Equation number	Equation	Missing quantity
2.7	$v_y = v_{0y} + a_y t$	$y - y_0$
2.10	$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$	v_y
2.11	$v_y^2 = v_{0y}^2 + 2a_y (y - y_0)$	t
2.9	$y - y_0 = \frac{1}{2}(v_{0y} + v_y)t$	a_y
	$y - y_0 = v_y t - \frac{1}{2} a_y t^2$	v_{0y}

Important: equations apply ONLY if acceleration is constant.

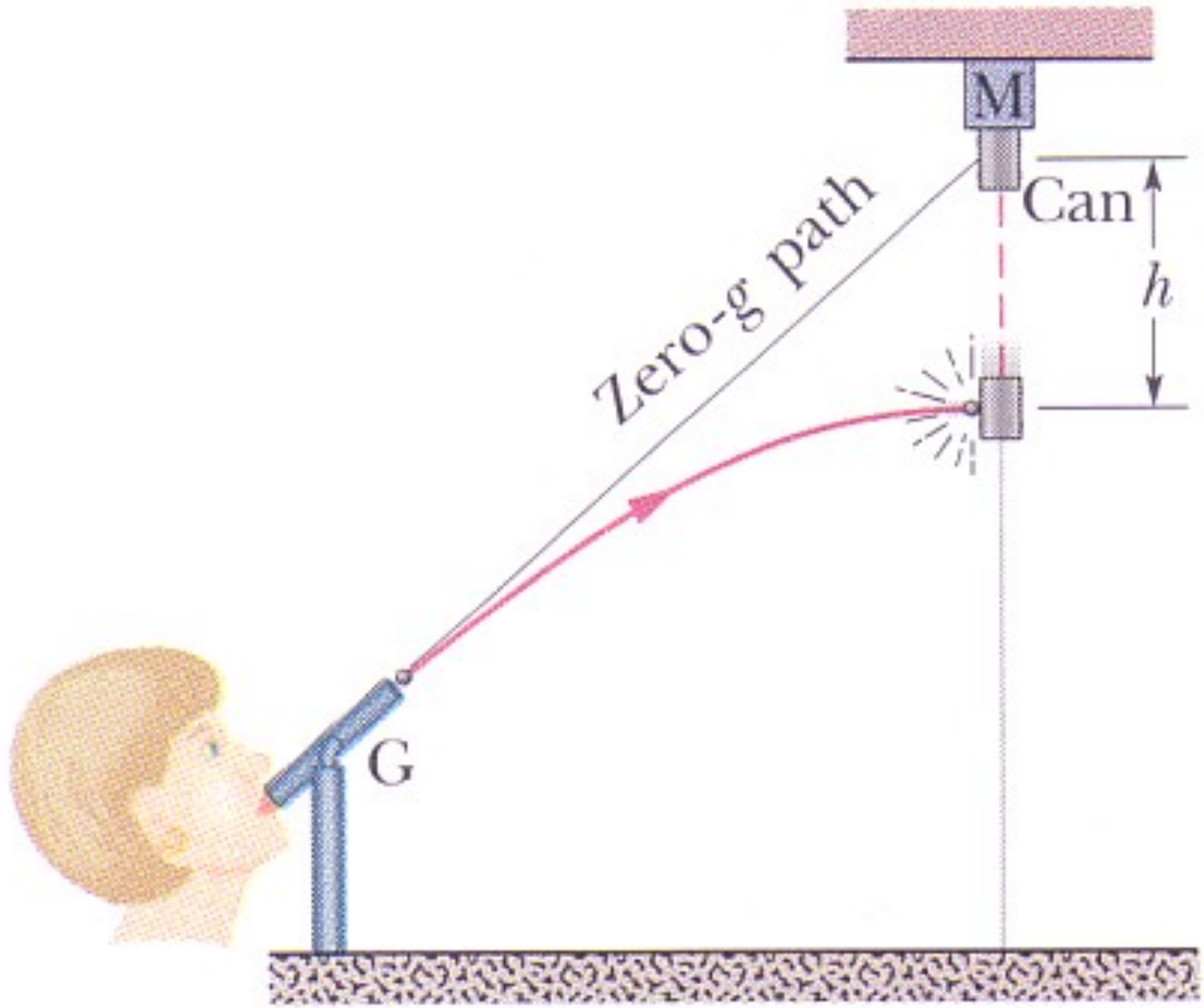
Equations of motion for constant acceleration

Special case of free-fall under gravity, $a_y = -g$.

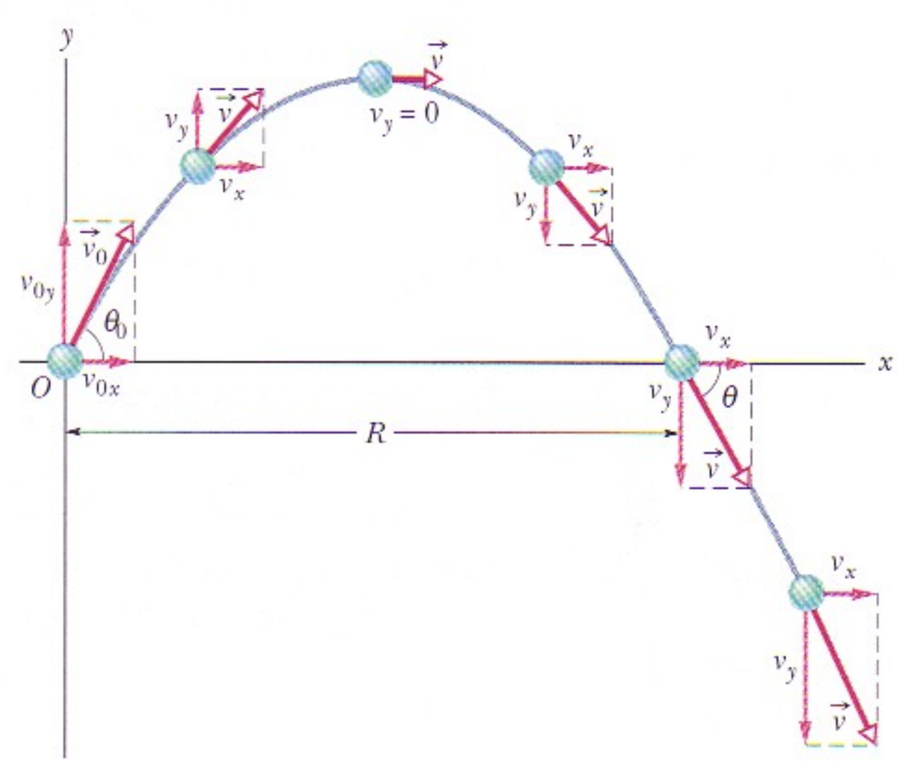
$g = 9.81\ \mathrm{m/s^2}$ here at the surface of the earth.

Equation number	Equation	Missing quantity
	$v_y = v_{0y} - gt$	$y - y_0$
	$y - y_0 = v_{0y}t - \frac{1}{2}gt^2$	v_y
	$v_y^2 = v_{0y}^2 - 2g(y - y_0)$	t
	$y - y_0 = \frac{1}{2}(v_{0y} + v_y)t$	a_y
	$y - y_0 = v_y t + \frac{1}{2}gt^2$	v_{0y}

Demonstration

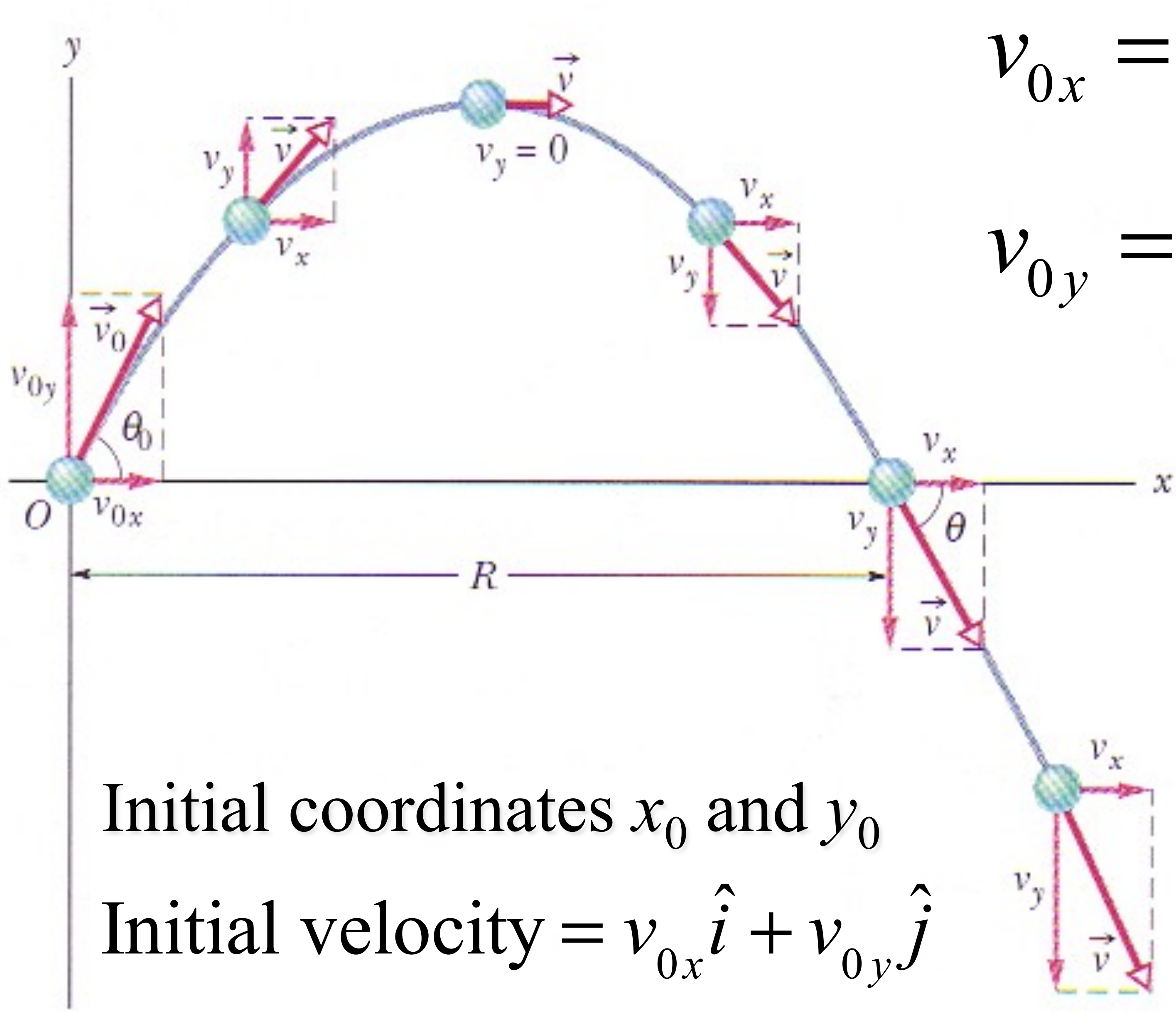


Projectile motion



- Motion in a vertical plane where the only influence is the constant acceleration due to gravity.
- In projectile motion, the horizontal motion and vertical motion are independent of each other, i.e. they do not affect each other.
- This feature allows us to break the motion into two separate one-dimensional problems: one for the horizontal motion; the other for the vertical motion.
- We will assume that air resistance has no effect.

Analyzing the motion



$$v_{0x} = v_0 \cos\theta_0$$

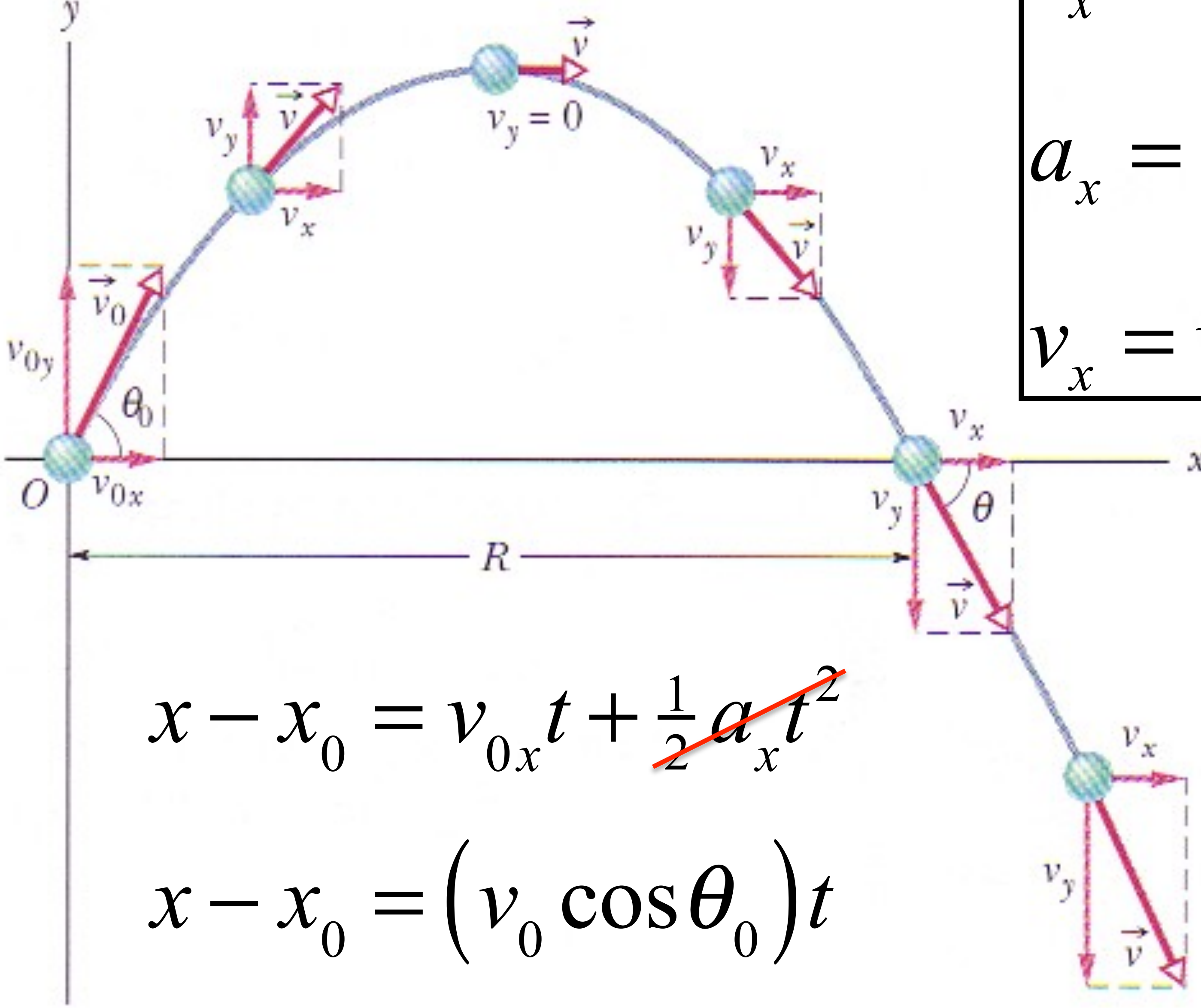
$$v_{0y} = v_0 \sin\theta_0$$

Initial coordinates x_0 and y_0

Initial velocity $= v_{0x}\hat{i} + v_{0y}\hat{j}$

Horizontal motion

$$\boxed{\begin{aligned} v_x &= v_{0x} + a_x t \\ a_x &= 0 \quad \Rightarrow \\ v_x &= v_0 \cos\theta_0 \end{aligned}}$$



$$x - x_0 = v_{0x} t + \cancel{\tfrac{1}{2} a_x t^2}$$

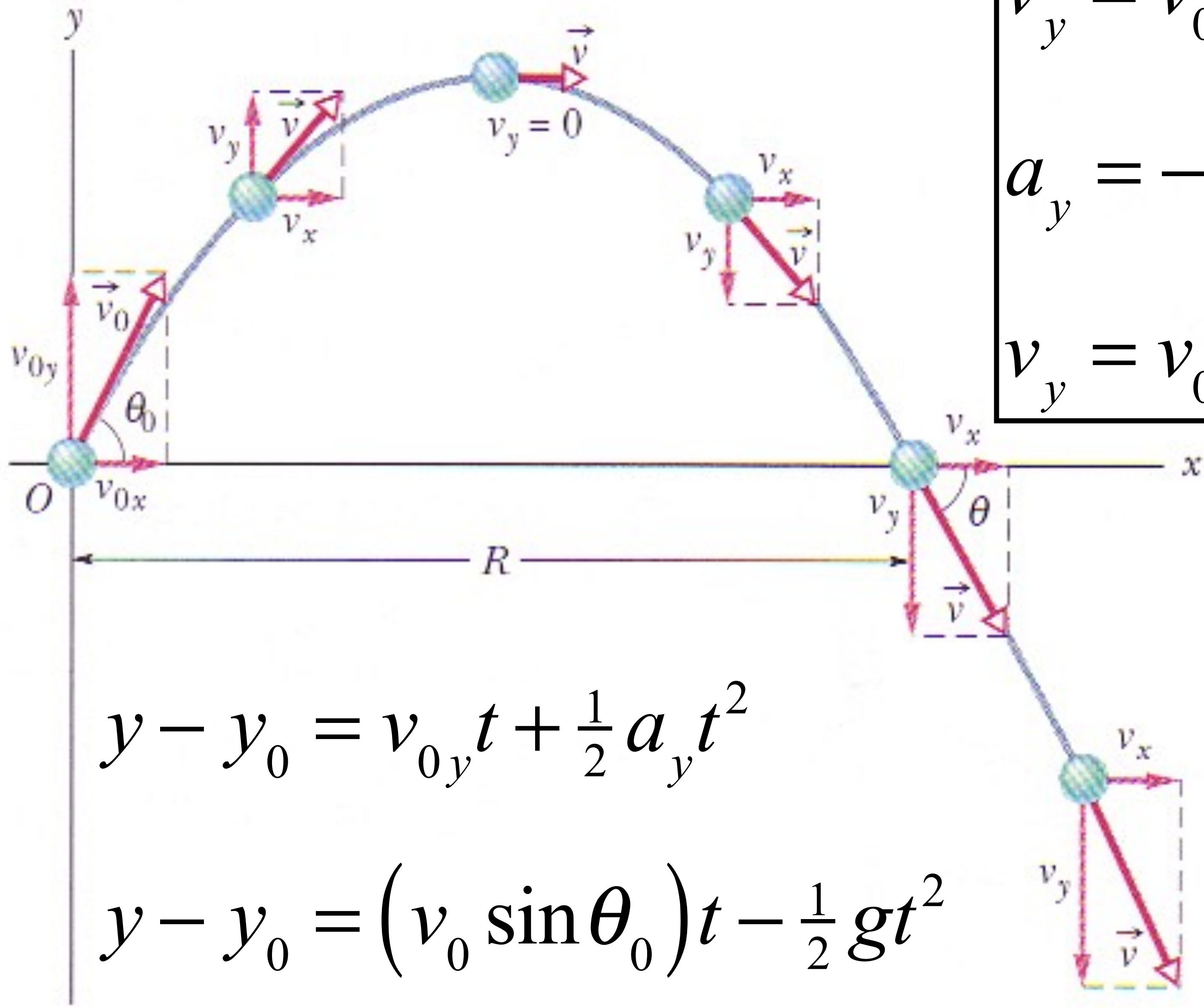
$$x - x_0 = \left(v_0 \cos\theta_0\right) t$$

Vertical motion

$$v_y = v_{0y} + a_y t$$

$$a_y = -g \quad \Rightarrow$$

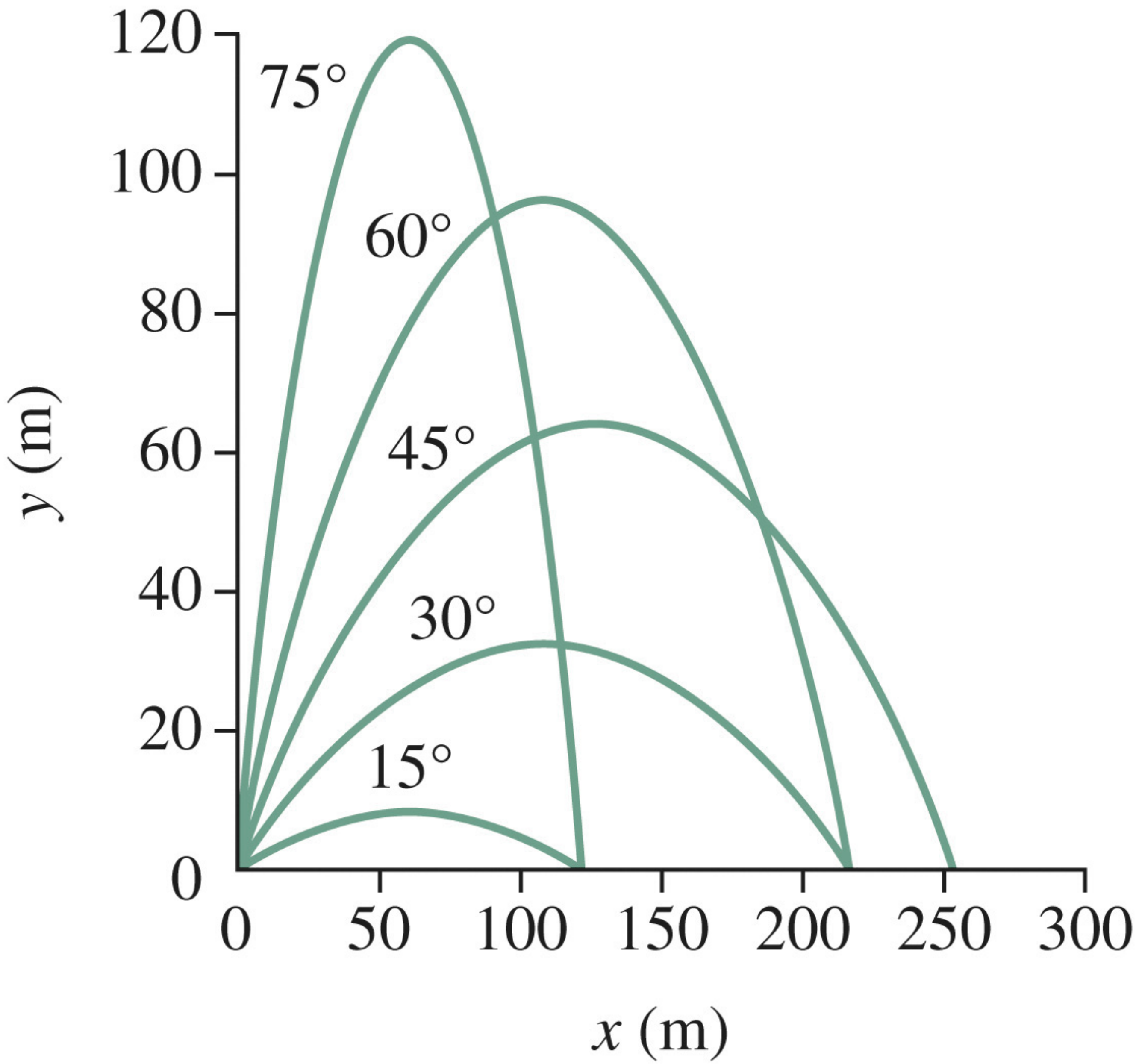
$$v_y = v_0 \sin\theta_0 - gt$$



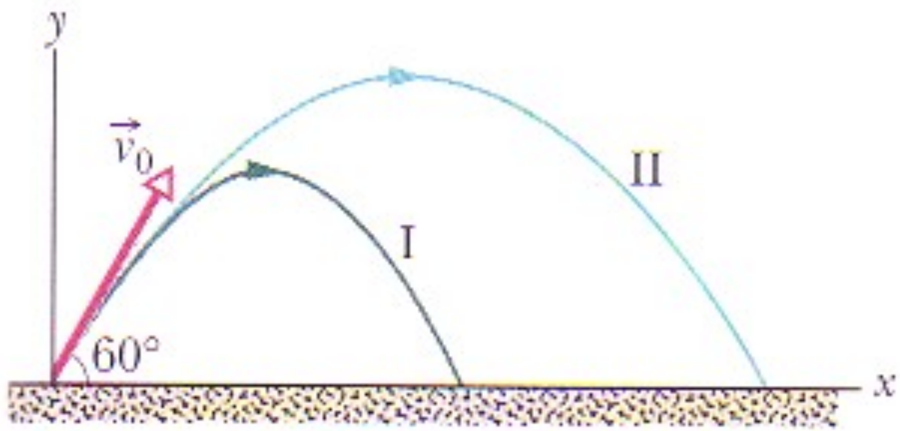
$$y - y_0 = v_{0y} t + \tfrac{1}{2} a_y t^2$$

$$y - y_0 = \left(v_0 \sin\theta_0\right) t - \tfrac{1}{2} g t^2$$

Maximum Range



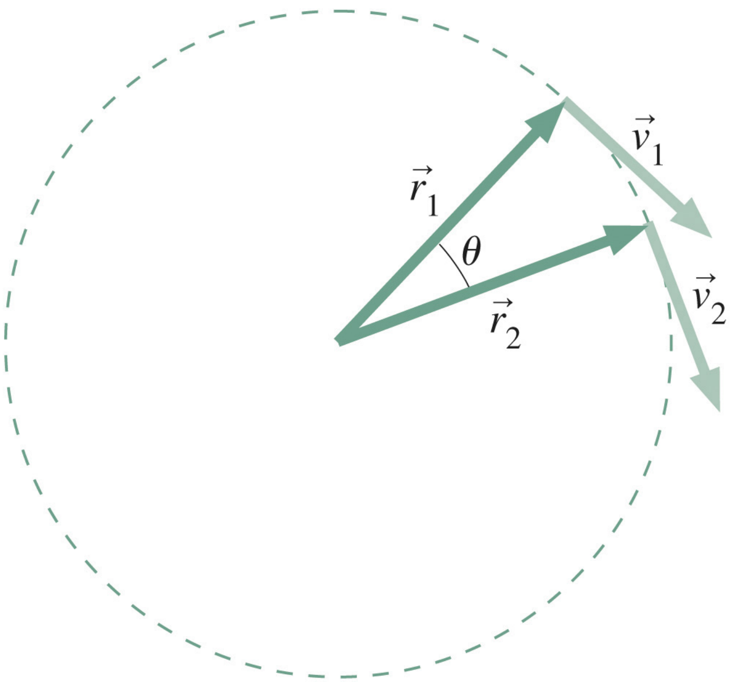
The effects of air



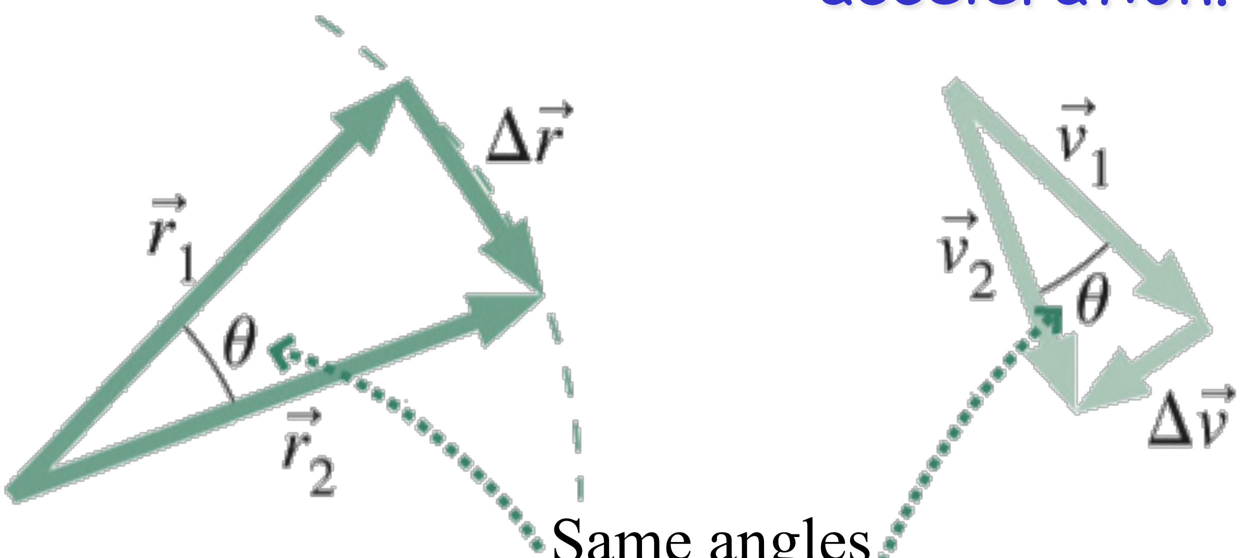
The physics professor's home run always goes further than the professional's

	Path I (Air)	Path II (Vacuum)
Range	98.5 m	177 m
Maximum height	53.0 m	76.8 m
Time of flight	6.6 s	7.9 s

Uniform circular motion



- Although the speed, v, does not change, the direction of the motion does, *i.e.*, the velocity, which is a vector, does change.
- Thus, there is an acceleration associated with the motion.
- We call this a centripetal acceleration.

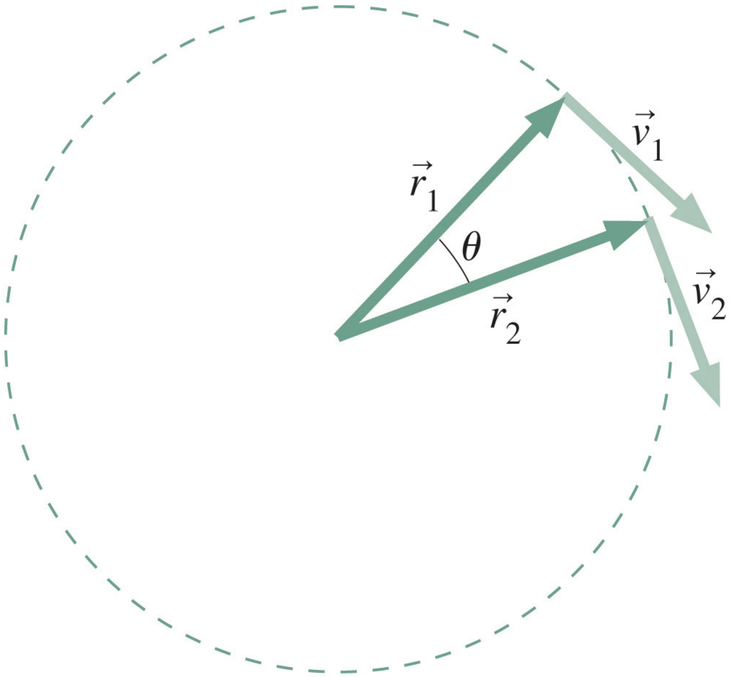


$$\Delta r = r\theta$$

$$\Delta v = v\theta$$

$$\Rightarrow \frac{\Delta v}{v} = \frac{\Delta r}{r} = \frac{v\Delta t}{r}$$

Uniform circular motion



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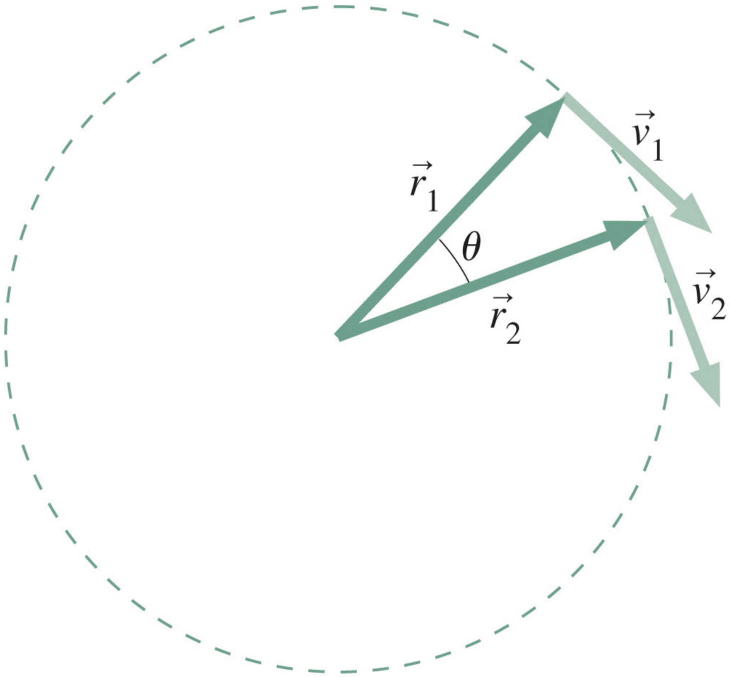
Centripetal acceleration:

$$a_c = \frac{v^2}{r} \quad \text{(uniform circular motion)}$$

- A vector that is always directed towards the center of the circular motion, i.e., it's direction changes constantly.

Uniform circular motion



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Centripetal acceleration:

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Period: $T = \dfrac{2\pi r}{v}$ (sec) Frequency: $f = \dfrac{1}{T} = \dfrac{1}{2\pi}\dfrac{v}{r}$ (sec^{-1})